Math 131B-1: Optional "Homework" 10

1. Do Apostol 12.9, 12.12, 12.14.
2. Let $f : \mathbb{R}^2 \to R$ be given by

$$f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} & (x, y \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

   Show that all the directional derivatives of f exist at $(0,0)$, but f is not differentiable.
3. Show that if $f : \mathbb{R}^n \to \mathbb{R}^m$ has $f'(x) = 0$ for all x, then f is constant. (Hint: Two points determine a line, and you can take a directional derivative along any line away from a point.)
4. Try to find a function $f : \mathbb{R}^2 \to R$ whose mixed partial derivatives $D_{2,1}f$ and $D_{1,2}f$ exist and are not equal. (Hint: By Clairaut's theorem, what properties can't these partial derivatives have?)